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MATHEMATICAL LIFE

Sergei Ivanovich Adian (on his 75th birthday)

One of the most prominent native mathematicians, Academician Sergei Ivanovich Adian [Adyan] was born on January 1, 1931 in the mountain village of Kushchi (Dashkesan District of the Azerbaijan Soviet Republic) situated forty kilometers from the city of Kirovabad (now Gjanja).

His father, Ivan Arakelovich Adiyan, was born the son of a shepherd in 1908. Not having the opportunity to finish secondary school, Ivan became a carpenter and worked at local building sites. In 1930 he married Lusik, the 17-year-old daughter of a local farmer, Konstantin Truziyan. Two years later Sergei's parents moved to Kirovabad, where Ivan worked as a carpenter. Initially they rented a room, and it was only at the end of the 1930s that the father bought a plot of land in the centre of the city and built a small house with one room, a porch, and a little cellar. Being a builder, Ivan planned to add a second floor to the house, but this plan was disrupted by the war. By that time there were already four children in the family. The mother did not work, but the parents completed their secondary

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education studies at an Armenian evening school for labourers. Although at the time Seryozha,[1] like his parents, did not speak Russian, he was sent in 1938 to study at the Russian secondary school no. 11 in Kirovabad. His father insisted on that, since he believed that after finishing the school it would be easier for his son to get a higher education. And so from his very first year in school the boy had to develop persistence and diligence. His problems with Russian were overcome already by the end of this first year.

In 1941, at the very beginning of the war, Sergei's father was conscripted. After a short training course somewhere in the northern Caucasus he was sent to the front. There he did not survive for long: due to incompetent leadership (perhaps even betrayal) of his commanders, his regiment was surrounded in the Crimean catacombs near Kerch, and his family received notice that he was missing. The mother got a job selling mineral water in a kiosk, and 10-year-old Sergei effectively became the head of the household, helping his mother keep house and bring up his two younger brothers Semik (8 years old) and Yurik (3) and his sister Svetlana (6).

There were good and, more important, exacting teachers in the school no. 11 attended by Sergei. His mathematical talent soon became apparent. Once, in the fourth grade, the teacher asked every student to solve one problem from a problem book, and she walked between the rows monitoring the progress. While everybody else was still working hard on the first problem, Sergei had already solved several of them. The teacher was pleased and went on with the experiment until the end of the lesson. As a result, Sergei solved 40 problems in one lesson. Another interesting episode took place in the tenth and last grade. Before the spring break, as part of preparation for final examinations, another teacher of mathematics, the school headmaster Sergei Arustamovich Ambartsumyan [Ambarzumian], gave his students homework in solid geometry based on trigonometric formulae from the popular problem book by Rybkin. The teacher asked everyone to solve only a couple of problems from each section, and he was immensely surprised when one of the students, Sergei Adian, handed him a thick notebook with complete solutions, drawings included, of all the problems from Rybkin's book! The teacher said that he would display this notebook in the school museum. It is not surprising that the Education Department of Kirovabad submitted to Baku, the capital of the Azerbaijan Republic, a petition to send Sergei Adian to the Moscow State University (MSU) to continue his education after completing his secondary school studies. But in Baku his name was crossed off the list. Following his school headmaster's recommendation, Sergei then went to Erevan with the purpose of entering Erevan University. But there they had a 'national politics' of their own. He was not allowed to register because prospective students were supposed to pass a written examination in Armenian, and the Armenian alphabet was certainly not a subject taught in a Russian school located in Azerbaijan. Finally, in 1948 Adian had to enter the Russian Pedagogical Institute of Erevan. His education there lasted only one year. After that he and others among the best students in Erevan were sent to continue their studies in Moscow. This action was organised by the USSR Ministry of Education, and every student was transferred to an institution similar to the

[1]Serezha is a Russian nickname for Sergei.

one he was taken from. As a consequence, Adian was refused admission to MSU, and this is how, for purely formal reasons, one more attempt of his to end up there failed. In his speech at a conference devoted to the 100th birthday of Lyudmila Vsevolodovna Keldysh, Adian commented on this story in the following words: "I should admit that at that time I was extremely lucky: I was not able to go to MSU. As fate willed, I went to the Moscow State Pedagogical Institute (MSPI), where I met Petr Sergeevich Novikov, and he introduced me to his wife Lyudmila Vsevolodovna…"

If ever there was an encounter that could be called fortunate, it was the meeting of Adian and his future teacher, mentor, and friend (in spite of the difference in age) P. S. Novikov. For a reader wishing to know more about this remarkable person, we recommend with pleasure the articles [50] and [51] written by Adian himself about his memories of his teacher. From [50] one can also learn about the wonderful and, unfortunately, now all too infrequent relationship between a student and his teacher exemplified by these two outstanding persons.

Adian started his research work at MSPI, with P. S. Novikov as his advisor, in the field of the descriptive theory of functions. In his first work as a student in 1950, he proved that the graph of a function $f(x)$ of a real variable satisfying the functional equation $f(x + y) = f(x) + f(y)$ and having discontinuities is dense in the plane. (Clearly, all continuous solutions of the equation are linear functions.) This result was not published at the time. It is curious that about 25 years later the American mathematician Edwin Hewitt from Seattle gave preprints of some of his papers to Adian during a visit to MSU, one of which was devoted to exactly the same result, which was published by Hewitt much later.

In his graduate work in 1953 relating to the theory of discontinuous functions, Adian constructed examples of semicontinuous functions on the interval $[0, 1]$ that, for any partition of the interval into a countable number of subsets E_i, have discontinuities on at least one of the subsets upon restriction to this subset. This contribution also was not published right away. In 1958, following a proposal of Adian, the work was published in the Scientific Notes of MSPI as joint work of two authors [8].

In the fall of 1954, P. S. Novikov suggested to Adian (then in his third year of graduate study) that he work on the word problem for finitely presented groups, noting that though Adian's results already obtained in the theory of functions were certainly enough for a Ph.D. thesis, this new problem was more interesting, was mentioned in A. G. Kurosh's monograph, and was a difficult problem that had resisted solution by Novikov's methods. In suggesting it, Novikov considered the fact that Adian had already mastered thoroughly the methods of Novikov's proof, not yet published, of the unsolvability of the word problem. By the beginning of 1955 Adian had managed to prove the undecidability of practically all non-trivial invariant group properties, including the undecidability of being isomorphic to a fixed group G, for any group G. These results made up his Ph.D. thesis, defended in 1955 and first published the same year as a brief note in *Doklady* [1] (the full text of the proof was published in [3] two years later). This is one of the most remarkable, beautiful, and general results in algorithmic group theory and is now known as the *Adian–Rabin theorem.*[2] We would like to present some details about it.

Suppose that we have a group G given by a finite number of generators and defining relations in the form

$$G = \langle a_1, \dots, a_m \mid r_1 = 1, \dots, r_n = 1 \rangle, \tag{1}$$

where the r_i are words in the 'group alphabet' $\{a_1, \dots, a_m, a_1^{-1}, \dots, a_m^{-1}\}$ (thus, G is the quotient group of the free group with generators $a_1, \dots, a_m$ by the normal subgroup generated by the elements $r_1, \dots, r_n$). Shortly before [1] appeared, P. S. Novikov had published a fundamental result (now classical) on the algorithmic unsolvability in general of the *word problem* [Nov1]. He was the first to construct an example of a group G for which it is impossible to find an algorithm deciding, for a given word r, whether the equality $r = 1$ is valid in the group.

However, from a general mathematical point of view it seems no less natural to ask about algorithmic decidability of properties of groups G *themselves* viewed as *abstract* algebraic objects. Non-trivial decidable properties do exist: an example is given by the property that $G/[G, G]$ contains an element of order 75. In this respect, the Adian–Rabin theorem looks all the more surprising, and it required considerable boldness just to suppose that something of this kind would be possible. Omitting technical details, the theorem asserts that exotic examples of this kind are, in essence, uniquely possible and that, for the overwhelming majority of 'natural' invariant group properties, the problem of their recognition is algorithmically unsolvable.

Of course, the history of mathematics offers quite a few other examples of work of undergraduate or graduate students which later became classical results in their fields. However, what distinguishes the first published work by Adian [1], [3] even in this brilliant company is its *completeness.* In spite of numerous attempts, nobody has added anything fundamentally new to the results during the past 50 years. It is not surprising that Adian's result was immediately used by A. A. Markov [Mar] in his proof of the algorithmic unsolvability of the classical problem of deciding when topological manifolds are homeomorphic. For this work Adian himself was awarded the Moscow Mathematical Society Prize in 1956 and the Chebyshev Prize of the Soviet Academy of Sciences in 1963. It should be noted that A. S. Esenin-Volpin, one of the official opponents of Adian's Ph.D. thesis, after reading and verifying the thesis, made a special trip to P. S. Novikov's dacha in the summer of 1955 to convince him that such work merited a D.Sc. degree. Novikov answered that there was nothing to be concerned about: he did not doubt that Adian would write another work for his D.Sc. dissertation. And the first opponent, A. I. Mal'tsev, proposed that the Academic Council of MSPI, where the defence was taking place, should call special attention to the outstanding level of the work.

After completing his graduate studies, Adian worked for several years (in close cooperation with P. S. Novikov) as an assistant professor in the Mathematical Analysis Department of MSPI. And in 1957 an event happened which completely changed the life of both him and his teacher. Namely, the Department of Mathematical Logic was created in the Steklov Mathematical Institute (MIAN), and

[2]The American mathematician Michael O. Rabin published a simpler proof of the result some years later.

P. S. Novikov was invited to lead it. Adian became one of the first members of this new department, and his subsequent research career was closely connected with it. Furthermore, the collaboration between Novikov and Adian on the Burnside problem started (about 1960) already within the precincts of MIAN.

Like several other great mathematical problems, the Burnside problem is striking in its deceptive simplicity. Fix $n \geqslant 2$ and consider the group

$$G = \langle a_1, \dots, a_m \mid X^n = 1 \rangle \tag{2}$$

with the identity $X^n = 1$ (a difference from (1) is that here the symbol X is not a fixed word but a variable running over the set of all words in the alphabet $\{a_1, \dots, a_m, a_1^{-1}, \dots, a_m^{-1}\}$). Is the group G finite?

In the monograph [ChM] devoted to combinatorial group theory, the Burnside problem is characterised as follows:

> *Very much like Fermat's Last Theorem in number theory, Burnside's problem has acted as a catalyst for research in group theory. The fascination exerted by a problem with an extremely simple formulation which then turns out to be extremely difficult has something irresistible about it to the mind of the mathematician.*

Before the work of Novikov and Adian an affirmative answer to the problem was known only for $n \in \{2, 3, 4, 6\}$ and the matrix groups. However, this did not hinder the belief in an affirmative answer for any period n. The only question was to find the right methods for proving it. As later developments showed, this belief was too naive. This just demonstrates that before their work nobody even came close to imagining the nature of the free Burnside group (2), or the extent to which subtle structures inevitably arose in any serious attempt to investigate it. In fact, there were no methods for proving inequalities in groups given by identities of the form $X^n = 1$.

An approach to solving the problem in the negative was first outlined by P. S. Novikov in his note [Nov2], which appeared in 1959. However, the concrete realization of his ideas encountered serious difficulties, and in 1960, at the insistence of Novikov and his wife Lyudmila Keldysh, Adian settled down to work on the Burnside problem. Completing the project took intensive efforts from both collaborators in the course of eight years, and in 1968 their famous paper [13] appeared, containing a negative solution of the problem for all *odd* periods $n \geqslant 4381$, and hence for all *multiples* of those odd integers as well.

The solution of the Burnside problem was certainly one of the most outstanding and deep mathematical results of the past century. At the same time, this result is one of the hardest theorems: just the inductive step of a complicated induction used in the proof took up a whole issue of volume 32 of *Izvestiya*, even lengthened by 30 pages. In many respects the work was literally carried to its conclusion by the exceptional persistence of Adian. In that regard it is worth recalling the words of Novikov, who said that he had never met a mathematician more 'penetrating' than Adian.

At the basis of the proof of the Novikov–Adian theorem was the idea, from the original note [Nov2], of bounding the infinite system of defining equations of a Burnside group, in order to apply Tartakovskii's theory of small cancellations.

This would enable one to deduce the infiniteness of the group from the existence of an infinite square-free Thue–Arshon sequence. However, the concrete realization of this approach required a large number of new ideas and constructions; of them we would like to mention the idea of a global classification with respect to the ranks of all periodic words of degree n, a representation of a Burnside group as the limit of an infinite chain of quotients of a free group, and also the notion of *cascade*. In essence, Novikov and Adian created a harmonious and logical theory for investigating the groups (2) themselves as well as similar 'Burnside-like' groups.

In contrast to the Adian–Rabin theorem (which, as noted above, distinguished itself by its completeness), the paper [13] in no way 'closed' the problem. Moreover, over a long period of more than ten years Adian continued to improve and simplify the method they had created and also to adapt the method for solving some other fundamental problems in group theory. By the beginning of 1980s, when other contributors appeared who mastered the Novikov–Adian method, the theory already represented a powerful method for constructing and investigating new groups (both periodic and non-periodic) with interesting properties prescribed.

For example, in the following year 1969, Adian used the theory developed to prove the independence of the system of group identities

$$(x^{pn}y^{pn}x^{-pn}y^{-pn})^n = 1,$$

where p runs over the set of all primes while $n \geqslant 1003$ is fixed. This means that any such identity cannot be deduced from the others. By these means, simple examples of group varieties with no finite basis were given for the first time; this led to a solution of the well-known finite basis problem stated by B. Neumann (see [17] and [18]). In the proof of this result, Adian applied a modification of the method in which not all periodic words are used but only those that satisfy some explicitly given conditions. Two more years later, Adian published [19], in which a new generalization of the Novikov–Adian method was introduced for investigating the central extension $A(m, n)$ of the Burnside group $B(m, n)$ by a cyclic group. He established that the intersection of any two non-trivial subgroups of $A(m, n)$ is infinite for arbitrary $m \geqslant 2$ and odd $n \geqslant 665$. The latter property characterises the additive group of rational numbers in the class of Abelian groups. Adian thereby answered in the affirmative a question posed long before by P. G. Kontorovich about the existence of non-Abelian analogues of the additive group of rational numbers. In contrast to the group of rationals, which is locally cyclic, the non-Abelian analogues so constructed can have any finite number of generators. It was shown later that adding one defining relation $d^n = 1$ to the group $A(m, n)$, where d generates the centre of the group, leads to a countable group admitting only the discrete topology. The question of the existence of such groups was known as A. A. Markov's problem.

All the results cited above were included in Adian's monograph [22], where the Novikov–Adian method was substantially simplified and used for a deeper investigation of the structure of Burnside groups. The bound $n \geqslant 4381$ was improved to $n \geqslant 665$ (to the present day nobody has been able to improve the latter estimate). It was proved in the monograph for the first time that free Burnside groups of odd period $n \geqslant 665$ are not only infinite but also have exponential growth, that is, the number of group elements represented by words of length k grows as an exponential a^k. Moreover, by increasing the period n one can make the base a arbitrarily close to the analogous base for the absolutely free group with the same number of generators. It was also proved in [22] that Burnside groups of sufficiently large odd exponent possess other important properties of absolutely free groups.

In [23], Adian introduced the notion of a *periodic product* of groups, which is associative and satisfies the condition of being hereditary with respect to subgroups. This led to a solution of a well-known problem of A. I. Mal'tsev. Several natural properties of the product were established. A criterion for periodic products of groups to be simple was later obtained which allowed one to construct infinite series of finitely generated infinite simple groups in varieties of periodic groups. It is proved in [31] that free periodic groups of sufficiently large odd exponent are not amenable, and hence symmetric random walks on them are non-recurrent.

An interesting and productive modification of the Novikov–Adian method was proposed by A. Yu. Ol'shanskii in 1982. It uses the geometric language of van Kampen diagrams instead of the language of formal deduction of group equations of the form $W = 1$ from defining relations [Ol1], [Ol3]. In Ol'shanskii's interpretation, the defining relations are added one per rank; this substantially simplifies the definitions but complicates the analysis of the newly added relations. In addition, the period of a group is increased by a factor of about 10^{10} (compare the estimates in [Lys] and [Iv]). Using his modification Ol'shanskii constructed for the first time examples of infinite periodic groups of prime period $p \geqslant 10^{75}$ all whose subgroups have order p (the so-called *Tarsky monsters* [Ol2]). The later paper [40] of Adian and his student I. G. Lysenok [Lysionok] demonstrates that such examples can be also constructed for any odd $n \geqslant 1003$ on the basis of the original technique of the monograph [22].

The most important and intriguing problem remaining unsolved after the papers of Novikov and Adian was the Burnside problem for sufficiently large periods of the form $n = 2^k$. This problem was independently solved in papers of S. V. Ivanov [Iv] and I. G. Lysenok [Lys]. As a consequence, they managed to prove that the groups $B(m, n)$ are infinite for all $n \geqslant$ some k; [Iv] proves this for $k = 2^{48}$, and [Lys] proves it for $k = 8000$. It turned out that a free Burnside group of even period n has a richer structure of finite subgroups than a Burnside group of odd period.

All of these theorems use the Novikov–Adian theory or its modifications in one way or another. In this area the fundamental open problem, which will presumably require essentially new ideas, is the case of a small period (for instance, $n = 5$ or $n = 8$). More details on the history of this question can be found in Adian's survey articles [32] and [53].

There is no need to describe how great the repercussions of the paper [13] have been. It is enough to mention that already in 1970 (that is, less than two years after its publication in Russian) Adian received an invitation to give a lecture at the International Congress of Mathematicians in Nice. And in 1999 these results were awarded a State Prize of the Russian Federation.

Among the other research interests of Adian, we would like to emphasize his continued activity involving algorithmic problems for semigroups with one defining relation. It is well known that for *groups* with one defining relation (that is, of the form (1) with $n = 1$) the word problem is solvable [Mag]. Quite unexpectedly, the analogous problem for semigroups (as well as some related problems) is much harder and has remained unsolved for more than half a century, although there is serious hope of finally obtaining an affirmative solution. Adian has worked on this very hard problem several times throughout his career; his most recent paper [55] on the subject was published last year. His partial results in this direction are among the best that have been obtained.

In 1965, at the invitation of A. A. Markov, Adian also took a second position, in the Department of Mathematical Logic at MSU. His work there continues to ensure a close and fruitful collaboration of the department with the Department of Mathematical Logic at MIAN.

In 1973, because of a serious illness of P. S. Novikov and at Novikov's personal request, Adian was appointed head of the department. A corresponding application was put in on the advice of Academician I. M. Vinogradov, who was the director of MIAN. This was not a simple time in the history of the department (and, perhaps, even in the whole history of mathematical logic in the USSR). Neither Adian nor his teacher nor many other academicians were members of the Communist Party of the Soviet Union, and there were other, Communist, candidates actively promoted for the post of head of the department. An older reader can easily imagine the exotic forms of this activity. The fact that common sense finally won out demonstrates again the high confidence and respect enjoyed by Adian already at that time. When one of the academicians proposed, during a meeting of the Scientific Council of MIAN, to discharge the doctoral candidate G. S. Makanin because he was taking so long to write a D.Sc. dissertation, Adian resolutely opposed the proposal and explained to the members of the council that Makanin was working on a very hard problem. The members trusted Adian's judgement, and a few years later Makanin defended a brilliant D.Sc. dissertation containing a proof of the algorithmic decidability of the solvability problem for equations in free semigroups. This result has now become classical.

Incidentally, the Department of Mathematical Logic in the Faculty of Mechanics and Mathematics at MSU went through a similar period of turbulence, for similar reasons, when the head of the department, Corresponding Member of the Soviet Academy of Sciences A. A. Markov, fell sick at the end of the 1970s. In many respects due to the energy, integrity, and diplomatic skills of Adian, this situation was also resolved favorably for the department. In particular, Adian managed to keep A. L. Semenov, who had just defended his Ph.D. dissertation, in his position at the department. In this case, Adian also had to guarantee that Semenov would get a D.Sc. degree. This prediction was also confirmed, not to mention that Semenov was one of the most active and helpful teachers in the department for many years. In this context another story is significant. At one time the Methodology Commission of the department tried to remove the obligatory lecture course "Introduction to Mathematical Logic" from the first-year curriculum. Adian learned about this when a projected new curriculum was submitted to the university administration. He immediately contacted everybody who might influence the outcome in the department, convinced them, and managed to withdraw the new curriculum and restore the obligatory first-year course in logic. This lecture course is delivered to the present day, indisputably broadening the horizons of the graduates of the Faculty of Mechanics and Mathematics. After the death of Markov in 1980, A. N. Kolmogorov was appointed head of the department and the situation became stabilized.

Adian has always devoted much attention to strengthening the Department of Mathematical Logic at MIAN, to training researchers in the Department of Mathematical Logic at MSU, and to developing new connections between these two related groups. He has had great success in this direction. Under his guidance more than thirty Ph.D. and D.Sc. dissertations have been written. His students are prominent researchers in algebra, mathematical logic, and computational complexity theory. After finishing at MSU, the strongest of them transferred to positions in the Department of Mathematical Logic at MIAN, which under his leadership became one of the most prominent and respected research centres in logic. His student A. A. Razborov is a well-known expert in computational complexity theory and a Nevanlinna Prize winner, and was elected a corresponding member of the Russian Academy of Sciences. Very recently L. D. Beklemishev, another student of Adian, was also elected a corresponding member.

In the Department of Mathematical Logic at MSU Adian has for many years led a seminar on algorithmic problems of algebra and logic, in addition to sharing leadership of the department's main seminar with V. A. Uspenskii. Several times he has also given mandatory lecture courses in mathematical logic for the first and fourth years, and special lecture courses on algorithmic problems of algebra and on infinite periodic groups. Adian is in essence the creator and leader of a whole research school in mathematical logic and algorithmic problems of algebra.

Besides his productive research and teaching activities, Adian is active in editorial and organizational work. As long ago as the end of 1950s, S. M. Nikol'skii invited Adian, at the suggestion of P. S. Novikov, to edit the section on mathematical logic in *Referativnyi Zhurnal*: *Matematika*, the Russian mathematical review journal, because there was then a huge backlog of articles to be reviewed. In the shortest possible time Adian rectified the situation there with respect to logic by mobilizing almost all his colleagues for the thankless task of writing reviews (for only a paltry fee). At about the same time, he drew attention to the fact that a remarkable textbook on mathematical logic written by P. S. Novikov had not been published, and that undergraduate and graduate students had to read a typescript. Novikov explained that the publishing house Fizmatgiz had rejected the manuscript because they had not liked the frequent use of the term *Hilbert formalism* in the preface: this was regarded as propaganda for a harmful bourgeois philosophical theory. Novikov planned to return the advance of his fee to the publishers. Adian told him that this was simply unacceptable and began to help in revising and editing the book. He declined Novikov's offer that he should be coauthor, and about half a year later the first edition of Novikov's textbook on mathematical logic appeared. The book was subsequently translated into several foreign languages.

Even the people closest to Adian would probably not be so bold as to call him an easy person to work with. Everybody who has ever dealt with him knows his adherence to principles and his uncompromising nature with respect to quite diverse questions, as well as his careful attention to details. However, those who have been in closer contact with him (and the authors of the present note belong to this list) well know another aspect. In the end it almost always happens, in some incomprehensible way, that Adian has in fact been right from the very beginning. And his arguments have been at least worth considering always, without exception.

Quite indicative of his nature is the well-known story about the awarding of a Ph.D. to A. V. Kuznetsov, who had had only six years of secondary school. He was a person of natural gifts, and S. A. Yanovskaya arranged for him to take part in the seminar on mathematical logic. He obtained interesting research results, and gave lectures at seminars. The senior colleagues (P. S. Novikov, A. I. Mal'tsev, A. A. Markov, and S. A. Yanovskaya) submitted a petition to the *Vysshaya Attestatsionnaya Komissiya* (VAK, the Higher Certification Commission) to award him a Ph.D. degree. But this was rejected by VAK on the grounds that the degree could be approved by the Specialized Scientific Council only after a defence. At this point Mal'tsev asked Adian to figure out how the question could be resolved. Although Novikov had doubts about success, Adian, confident in the triumph of justice, took up the matter with his usual persistence and stubbornness. In spite of the passive resistance of Kuznetsov himself, Adian first arranged for VAK to let Kuznetsov take the required (Ph.D.) exams and defend his dissertation. Then Adian organised the examination of Kuznetsov as well as the preparation of the text and the defence of the Ph.D. dissertation in MIAN. Adian himself was the first official opponent for that defence. Soon afterwards, Kuznetsov was invited to Kishinev (now Chişinău) for the post of head of the Laboratory of Mathematical Logic in the Mathematical Institute of Moldavia. There he has prepared many students, of whom three have defended D.Sc. dissertations.

For many years Adian was the head of the Specialized Scientific Council of VAK in MIAN concerned with defence of D.Sc. dissertations in mathematical logic, algebra, number theory, geometry, and topology, first as the vice-chairman and later, after the decease of Academician I. M. Vinogradov, as the chairman. In 1991 he asked to be relieved of the chairman position in view of his 60th birthday. However, when the directorate of MIAN proposed for this post a person who manifestly was not suitable, Adian could not reconcile himself with this and declared that he was prepared to remain in the position until a more appropriate successor was proposed. He was supported in this question by the head of the Department of Geometry and Topology, S. P. Novikov. Since a new candidate had to pass the decision of the Division of Mathematics of the Russian Academy of Sciences, the academic secretary of the Division of Mathematics of RAS, A. A. Gonchar, had to intercede and lead a discussion of possible candidates among the heads of the departments for all the corresponding areas of specialization. Finally, a candidate was chosen who was unanimously supported by the heads and the Division of Mathematics, and he was confirmed by VAK.

Of course, such stands in life brought a lot of troubles to Adian (the lateness in his being elected a member of the Academy was mostly due to his having such a 'high profile') and helped him acquire a lot of enemies. However, the same strains of character have allowed him to acquire true friends among people of like mind who are impressed by his directness and open temperament.

Adian did not wish to recognise government restrictions on human relations as necessary even at a time when this was fraught with various risks. During the ICM in Nice (1970) he met the American mathematician (of Sicilian origin) Frank B. Cannonito, and their friendship, which lasted from that moment until the beginning of Perestroika, was directed at the *development and broadening of friendly relations between mathematicians of the USSR and the USA*, *in spite of efforts of the* *authorities of both countries to kindle the cold war*, as Adian himself said. Besides Cannonito, Adian has kept in touch with many outstanding foreign mathematicians over the course of several decades. Among them are W. W. Boone (Urbana, IL), A. Tarsky (Berkeley), K. Gödel (Princeton), M. Hall (Los Angeles), W. Magnus (New York), G. Baumslag (New York), D. Solitar (Toronto), S. Maclane (Chicago), A. Nerode (Cornell University), G. Higman (Oxford), B. Neumann (Canberra), O. Kegel (Freiburg), J. Mennicke (Bielefeld), and others.

There are many people in Russia and the Commonwealth of Independent States who are profoundly grateful to Adian for everything he has done and continues to do for them. First and foremost among these people are his students (and also many students of his colleagues as well as some young hopefuls). We all know well that he is always nearby and ready to help in word and deed. In this respect we ought to recall once again the story about the actual rescue of the Division of Logic in the beginning of the 1970s: a different result would have been painful to many people. Also worth mentioning is his activity on the Council of Experts of VAK. At a time when this body was considered by many to be an instrument of the (Communist) Party's influence on science, the honest and uncompromising vote of Adian supported many talented scientists. He was one of the most active members of the Commission on Mathematical Education in the Division of Mathematics of the Soviet Academy of Sciences since the commission was founded under the chairmanship first of Academician I. M. Vinogradov and later of Academician L. S. Pontryagin. An important role here was played by his pedagogical education in MSPI, where he did practical teaching work guided by the well-known methodologist E. S. Berezanskaya. The older generation remembers well her famous mathematics textbook for the fifth grade; with the help of this book, the children of the whole country were learning logical thinking at the most appropriate age.

Adian has three adult children, two daughters and one son. His son Ivan graduated from the Faculty of Mechanics and Mathematics at MSU. The older daughter Vera graduated with honours from the Faculty of Philology of MSU and in recent years has taught Russian on contract in London. The younger daughter Lena graduated from the S. G. Stroganov Moscow State University of Arts and Industrial Design (Ceramics Department). She does painting and ceramic arts and has displayed her works often at the Central House of Artists and at exhibitions of young Russian artists; very recently she was elected a member of the Union of Artists of the Russian Federation.

We—his students, friends, and colleagues—congratulate Sergei Ivanovich Adian once again on his birthday and wish him excellent health, good luck, and further creative successes for the good of Russian and worldwide science.

L. D. Beklemishev, *I. G. Lysenok*, *A. A. Mal'tsev*, *S. P. Novikov*,
M. R. Pentus, *A. A. Razborov*, *A. L. Semenov*, *V. A. Uspenskii*

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